Mathematics

Math at its core is about establishing truths separate from sensual qualities, seeking patterns based upon these truths, systematically removing contradictions/inconsistencies from the patterns, and formulating conjectures with all of the above in mind. It is the one true language apart from reality which makes it ironic that it is so useful.

Here we wish to provide resources which will help you to develop mathematical skills for where ever you choose to apply them.

Contents [show]

Precalculus Edit

Note that any autodidactic education requires a minimum amount of fundamentals, and to grasp the higher levels of math you absolutely need to understand the basic concepts known as precalculus, which is generally the math you will see up to high school. If you lack any of these fundamentals, you should refresh your knowledge at pages like Khan Academy or PatrickJMT. If you think you are fit, you can also directly start with calculus, although I would advise to skim a Precalculus book before you do so.

If you totally forgot everything or are a beginner, it is recommended you do the interactive exercises on Khan Academy because they are really helpful tools to quickly refresh your school knowledge up until calculus. You should do all the chapters up to Precalculus, that is: Early Math, Arithmetic, Pre-Algebra, Basic Geometry, Algebra I, Geometry, Algebra II, Trigonometry, Probability and Statistics. You don't need to listen to every video, but you should cover each exercise once to check if you understand it. Once you finish the Precalculus module, you can continue with your first book.

If you are still fit regarding math, you should at least do the Precalculus module on Khan Academy to be sure you have grasped everything necessary. For a general overview on the topics to come, choose any book on Precalculus, though I recommend one of the following:

- Simmons' Precalculus Mathematics in a Nutshell: Geometry, Algebra, Trigonometry (Concise refresher)
- Stewart's Precalculus: Mathematics for Calculus
- Cohen's Precalculus with Unit Circle Trigonometry
- Stitz & Zeager's Precalculus (Free but might have a bit too much extra material)

You can finish Stewart's book in a few weeks. It is already structured in a way that you can do 1-2 chapters per day for 6 days and do a review day on the 7th. You will be familiar with most concepts in this book, but especially if you just come out of High School or have just finished Khan Academy from zero, it will be a good exercise for you.

Those of you who've never done Precalculus, Trigonometry, or even Algebra 2 in high school or feel like you've forgotten all of it might be thinking that you should start with an Algebra 2/3, College/Intermediate Algebra, or Trigonometry book before starting a Precalculus book since that's how it worked in high school, but don't be. If you take a look at Stewart's "Algebra and Trigonometry" book, all the content is mostly identical to his Precalculus book. Similarly, Stewart's "College Algebra" and "Trigonometry" books also just recycle the corresponding material in his Precalculus book.

Basic Axiomatic Geometry Edit

Disclaimer: You don't really need to learn this stuff anymore unless you want to. The Synthetic Geometry (aka Axiomatic or Pure Geometry) these books use isn't how people usually go about solving geometry problems. They use Analytic Geometry (better named Coordinate Geometry) and solve problems by reducing them into algebra, trigonometry, and calculus problems. The axiomatic approach stuck around beyond the invention of coordinate geometry and calculus as a means to teach proofs and rigorous reasoning but that too has been replaced by Intro to Proofs (or Discrete Math) classes based on naive set theory and logic.

- Kiselev's Geometry: Book I. Planimetry & Geometry: Book II. Stereometry (Euclid's Elements distilled)
- Basic Geometry by Birkhoff and Beatley (Uses Birkhoff's axioms rather than the ones Euclid chose)
- Geometry: A High School Course by Lang and Murrow (A modern approach to rigorous geometry that covers somewhat different topics than other books do and isn't afraid to use coordinate)

Some advanced 2nd course material:

- Geometry Revisited by Coxeter and Greitzer
- Advanced Euclidean Geometry by Posamentier
- Advanced Euclidean Geometry (Dover Books) by Johnson
- College Geometry: An Introduction to the Modern Geometry of the Triangle and the Circle (Dover Books) by Altshiller-Court

Elementary Algebra Edit

- Elements of Algebra by Euler (Don't let its age discourage you, it's one of the best books ever written on elementary algebra even 250~ years later. Minor errata: Euler makes a small mistake when defining division and multiplication by $\sqrt{-1}$. See this review. He writes that $1/\sqrt{-1} = \sqrt{(1/-1)} = \sqrt{-1}$ but this is incorrect as $1 = \sqrt{-1}/\sqrt{-1} = \sqrt{-1}\sqrt{-1} = (\sqrt{-1})^2 = -1$. Instead multiply by $1=\sqrt{-1}/\sqrt{-1}$ to simplify it, $1/\sqrt{-1} = 1/\sqrt{-1} * \sqrt{-1}/\sqrt{-1} = \sqrt{-1}/(\sqrt{-1})^2 = -\sqrt{-1}$. This may or may not be pointed out depending on your translation.)

- Elementary Algebra for Schools by Knight and Hall [Archive.org for the 1896 edition, Amazon for the 1906 edition]

Some advanced "honors" books that cover extra material that hasn't been covered in algebra/public school courses for a century:

- Algebra : An Elementary Text-Book - For the Higher classes of Secondary Schools and Colleges Volumes 1 & 2 by Chrystal
- Higher Algebra: a Sequel to Elementary Algebra for Schools by Hall and Knight

Vector Geometry Edit

These books are the Linear Algebra approach to Geometry. Many of the topics covered here are scattered throughout other courses so they're not required reading to progress. The material is collected & organized together here for beginners who want to see the modern approach geometric problems or for someone who wants to revise his geometric skills.

- A Vector Space Approach to Geometry (Dover Books on Mathematics) by Melvin Hausner
- Elementary Vector Geometry (Dover Books on Mathematics) by Seymour Schuster
- Vector Geometry (Dover Books on Mathematics) by Gilbert de B. Robinson

Problem books Edit

These elementary problem books are meant for additional non-routine practice, challenges & puzzles, killing time, preparing for school competitions and exams, or to steal interesting questions from when you're teaching or tutoring students.

- Challenging Problems in Algebra (Dover Books) by Posamentier
- Challenging Problems in Geometry (Dover Books) by Posamentier
- The USSR Olympiad Problem Book: Selected Problems and Theorems of Elementary Mathematics (Dover Books) by Shklarsky, Chentzov, and Yaglom
- 103 Trigonometry Problems: From the Training of the USA IMO Team by Andreescu and Feng
- 104 Number Theory Problems: From the Training of the USA IMO Team by Andreescu
- 105 Algebra Problems from the AwesomeMath Summer Program by Andreescu
- 106 Geometry Problems from the AwesomeMath Summer Program by Andreescu
- 107 Geometry Problems from the Awesomemath Year-Round Program by Andreescu
- 108 Algebra Problems from the Awesomemath Year-Round Program by Andreescu
- "Problems From the Book" and "Straight From the Book" by Andreescu
- The Stanford Mathematics Problem Book (Dover Books) by Polya and Kilpatrick
- Sequences, Combinations, Limits (Dover Books) by Gelfand, Gerver, Kirillov, Konstantinov, and Kushnirenko
- Challenging Mathematical Problems With Elementary Solutions (Dover Books) by A.M. Yaglom and I.M. Yaglom
- Hungarian Problem Book I-IV containing the Eötvös Mathematical Competitions from 1894–1963

The following have more advanced problems at the university level up to preparing for qualifying exams during graduate school

- The Green Book of Mathematical Problems (Dover Books) by Hardy and Williams
- The Red Book of Mathematical Problems (Dover Books) by Williams and Hardy
- William Lowell Putnam Mathematical Competition: Problems & Solutions: 1938-1964
- The William Lowell Putnam Mathematical Competition: Problems and Solutions 1965–1984
- The William Lowell Putnam Mathematical Competition 1985-2000: Problems, Solutions and Commentary
- Contests in Higher Mathematics: Miklos Schweitzer Competitions, 1962-1991 by Szekely
- Problems in Mathematical Analysis I: Real Numbers, Sequences and Series by Kaczor and Nowak
- Problems in Mathematical Analysis II: Continuity and Differentiation by Kaczor and Nowak
- Problems in Mathematical Analysis III Integration by Kaczor and Nowak
- A Collection of Problems on Complex Analysis (Dover Books) by Volkovyskii, Lunts, and Aramanovich
- Problems in Group Theory (Dover Books) by Dixon
- Problems and Theorems in Analysis I: Series, Integral Calculus, Theory of Functions by George Polya and Gabor Szegö
- Problems and Theorems in Analysis II: Theory of Functions, Zeros, Polynomials, Determinants, Number Theory, Geometry by George Polya and Gabor Szegö
- A Hilbert Space Problem Book by Halmos
- Berkeley Problems in Mathematics by Paulo Ney de Souza and Jorge-Nuno Silva
- Problems and Solutions in Mathematics (Major American Universities PH.D. Qualifying Questions and Solutions)

Problem Solving and Heuristics Edit

Some strategies on how to approach difficult problems to solve them exactly or heuristically and dealing with Fermi problems :

- How to Solve It: A New Aspect of Mathematical Method by Polya
- Guesstimation: Solving the World's Problems on the Back of a Cocktail Napkin by Weinstein and Adam
- Guesstimation 2.0: Solving Today's Problems on the Back of a Napkin by Weinstein and Edwards
- Street-Fighting Mathematics: The Art of Educated Guessing and Opportunistic Problem Solving by Mahajan
- The Art of Insight in Science and Engineering: Mastering Complexity by Mahajan
- How to Solve It: Modern Heuristics by Michalewicz and Fogel (Heuristics focusing on CS/optimization problems)
- Problem-Solving Through Problems by Larson
- Putnam and Beyond by Gelca and Andreescu

Overview of Mathematics Edit

- What Is Mathematics? An Elementary Approach to Ideas and Methods by Richard Courant and Herbert Robbins
- Prelude to Mathematics (Dover Books) by Sawyer
- Concepts of Modern Mathematics (Dover Books) by Ian Stewart
- Mathematics: Its Content, Methods and Meaning (Dover Books) by Aleksandrov, Kolmogorov, Lavrentev, Sobolev, Gel'fand, et al.

Calculus Edit

Calculus is the study of change (derivatives) and accumulation (integrals) of functions. These topics are linked by the Fundamental Theorem of Calculus which states informally that "the accumulation of the changes of a function" and "the change of the accumulation of a function" result in the original function. The rest of calculus is just spamming these ideas on various applications and situations.

Some students struggle with calculus but honestly it is a really straight forward subject, especially compared to other advanced subjects in math. As long as you pay attention and go in trying to learn, you should quickly end up joining the rest of the math students in calling it 'trivial' in retrospect. Don't set yourself up to failure by thinking you're not capable of learning it because it will all click once you look at it right.

Single Variable Calculus Edit

The standards texts (Stewart, Rogawski, et al.) you see required for college classes are, in all honesty, quite terrible since they are not written with self-study in mind but just as a collection of exercises and a review of the basic methods. *Do* use them to practice calculus but not as a means to learn it.

For a well done intuitive approach using infinitesimals, which is the way everyone ends up thinking about calculus which is also technically *nonstandard* but by no means mathematically incorrect, "Elementary Calculus: An Infinitesimal Approach" by Jerome Keisler is a fantastic and free public domain book (also available in an inexpensive Dover paperback edition). If the infinitesimal approach intrigues you but you've already done a course in calculus or currently reading through another 1000 page book on calculus, Infinitesimal Calculus (Dover Books) by Henle and Kleinberg is a nice short 144 page book that develops the theory of infinitesimals in calculus in an accessible and clear manner. As you can probably infer from the page count, Henle's book doesn't have any material on the applications of calculus so don't use it as a standalone book to learn all of calculus from but as a supplement to see a different approach in understanding the subject the way it was originally invented.

For a rigorous *standard* (δ-ε) approach to the subject, your options are "Calculus" by Spivak or the classic "Calculus, Vol. 1: One-Variable Calculus, with an Introduction to Linear Algebra" by Apostol. Spivak's writing certainly has its fans but it sadly lacks much of the applications and motivation (related rates, optimization) that are standard in calculus making it hard to use on its own. Apostol's Calculus doesn't have this oversight and it's probably the best one to learn the material from on your own. Another rigorous option that also has copious amounts of physics applications, motivation, and intuition presented at the same time is "Introduction to Calculus and Analysis, Volume I" by Richard Courant and Fritz John. The book is a modern rewrite of the classic "Differential and Integral Calculus" by Richard Courant and includes the most material of the three and its exercise are the most difficult (perhaps a bit too difficult in places).

Other calculus books leave out the involved proofs, which you'll see later in analysis, and focus on conceptual understanding and applying calculus to accommodate weaker students or students that are less prepared in rigorous/abstract mathematics. This isn't a completely bad way of learning calculus but you might be annoyed by the occasional lack of explanation/justification in some isolated places. Some good books in this category are "Calculus: An Intuitive and Physical Approach" (Dover Books) by Kline, "Calculus With Analytic Geometry" by Simmons (contains a lot on history of the subject and its applications in physics and science), and "A First Course in Calculus" by Lang.

To just learn the methods of calculus, there are plenty of lectures on calculus for you to choose from on YouTube, such as 3Blue1Brown's intuitive *Essence of Calculus* series, and the videos of Martin Van Biezen, Sal Khan, PatrickJMT

, and blackpenredpen

Multivariable and Vector Calculus Edit

Again, the usual suspects you'll find assigned in college courses tend to make good exercise books but terrible introductions to the subject. Your options are the latter part of Keisler's book above for an infinitesimals approach; Lang's "Calculus of Several Variables" or the latter part of Simmons' book to continue with their approach for weaker and less prepared students; and "Calculus, Vol. 2: Multi-Variable Calculus and Linear Algebra with Applications to Differential Equations and Probability" by Apostol, or "Introduction to Calculus and Analysis, Vol. II" by Richard Courant and Fritz John (the paperback is split into 2 parts) to continue on with the standard rigorous approach.

The following texts take a slightly more rigorous approach than Apostol or Courant and go a bit deeper into the subject by covering differential forms and manifolds. Most single semester courses on vector calculus do not have time to reasonably cover this material, and consequently is usually skipped until later, but this advanced perspective can greatly aid one's understanding of the subject. You could study this material either when you first learn multivariable calculus or when you want a second pass on the subject, after just learning the basic methods, to improve your understanding while deepening your knowledge by generalizing what you've seen before. They can also be used as supplements or stepping stone to an advanced multivariable analysis course.

- C. H. Edwards Jr.'s Advanced Calculus of Several Variables (Dover Books)
- Hubbard and Hubbard's Vector Calculus, Linear Algebra and Differential Forms: A Unified Approach
- Harold M. Edwards' Advanced Calculus: A Differential Forms Approach
- Sternberg and Loomis' Advanced Calculus (for the utterly fearless)

Curves and Surfaces in $\mathbb{R}^2$ and $\mathbb{R}^3$ Edit

This subject is the study of Geometry using the tools that you learned in Vector Calculus and serves as a preparation to more abstract approaches to Differential Geometry you'll see in the future. Most schools only quickly pass through the subject during multivariable calculus but it will help in the long run if you study the material early on.

- Pressley's Elementary Differential Geometry
- do Carmo's Differential Geometry of Curves and Surfaces (Dover Books)

Ordinary Differential Equations Edit

The standard text used in college courses is "Elementary Differential Equations" by Boyce and DiPrima, which many people do seem to like (not me however). A cheap and very good alternative is "Ordinary Differential Equations" by Tenenbaum & Pollard (published by Dover) which is perfect for self study. Other well written books are "Differential Equations with Applications and Historical Notes" by Simmons and "Differential Equations" by Ross.

- Coddington's An Introduction to Ordinary Differential Equations (Dover Books)
- Arnold's Ordinary Differential Equations (advanced)

Linear Algebra Edit

When speaking of Linear Algebra, people refer to one of 2 complementary but different subjects: Matrix Algebra/Computational Linear Algebra and Theoretical Linear Algebra/Finite Vector Space Theory. Oddly enough, you could study them in any order but canonically you're typically expected to learn some matrix algebra first and then transition to vector spaces and/or more applied/numerical topics second. The necessary prerequisite knowledge is just precalculus but some calculus knowledge is useful and may appear in a few examples.

Matrix Algebra Edit

For a first exposure to the subject there really isn't that much to learn. You typically cover systems of equations, matrix operations, Gaussian elimination (also known as row reduction), LU decomposition, determinants, eigenvectors and eigenvalues, and diagonalization possibly with a few additional fluff subjects to round out a whole course. Many times you can pick up this material while studying calculus or ODEs (like with Apostol or Hubbard^2's book) so you can just skip to more advanced material. Also, the introductory material in first few chapters of advanced textbooks are often good enough to learn matrix algebra from if you're in a rush.

But while some students seem to inhale these topics and quickly move on, others will need to take their time before operating with matrices becomes natural to them. A gentle introduction for learners with weaker math skills can be found in "Matrices and Linear Algebra" by Schneider and Barker. Learners with slightly better math abilities can benefit more from "Matrices and Linear Transformations" by Cullen which is aimed at STEM students and contains extra material at the end on advanced material. A free book for students seeking a honors introduction to linear algebra (and basic proofs) is "Linear Algebra Done Wrong" by Treil (Don't worry, the title is a pun on Axler's "done right" book below). Another popular free book is Hefferon's Linear Algebra. There's also a whole host of vulgarly over expensive textbooks used by college courses at this level (like Strang's Introduction to Linear Algebra, Lay's Linear Algebra and Its Applications, Friedberg's Elementary Linear Algebra, etc) but most of them aren't very good and even if they were, the first 2 aforementioned books above are far cheaper thanks to them being published by Dover and the last 2 are free. If you prefer learning from videos, you can get a very solid intro using the linear algebra playlists on Khan Academy, PatrickJMT, or Engineer4Free. Bonus is they are all free.

Applied Linear Algebra Edit

For a first book in applied linear algebra, "Linear Algebra and Its Applications" by Strang is the standard text used but it is one of those love it or hate it texts. If you fall into the hate it camp, then Meyer's "Matrix Analysis and Applied Linear Algebra" is a good alternative. After reading one of them, you'll be more than ready to move onto advanced Numerical Linear Algebra and Matrix Analysis textbooks.

Finite Vector Spaces Edit

To get started on the theoretical side of linear algebra you obviously should be familiar with the basics of proofs. Once you are, theory side has a lot of classic and well loved textbook to choose from:

- Linear Algebra by Shilov (Dover Books)
- Finite Dimensional Vector Spaces by Halmos (Dover Books)
- Linear Algebra by Friedberg, Insel, and Spence
- Linear Algebra by Hoffman and Kunze

Of course there's also "Linear Algebra done Right" by Axler and on the one hand, the stuff he does is great... but on the other hand, he fucking HATES determinants and goes crazy avoiding them. Because of that you shouldn't use his book alone to learn from and you really should read Shilov alongside of it. But Axler certainly gives an unique development of the subject.

Refreshers and Advanced Books in LA Edit

Now if you want a challenge, start off with "Linear Algebra and Its Applications" by Lax. It is good for learning linear algebra for the first time if you're a hot shot freshman, using it as a second book on linear algebra, or as a 3rd refresher book for those who are entering graduate school. Another good 3rd book for deeper linear algebra study, and if you have the abstract algebra background for it, is Roman's "Advanced Linear Algebra". "Abstract Linear Algebra" by Curtis is a concise 162 page book that builds up the core of linear algebra from the beginning in a general abstract way and ends with Hurwitz's Theorem as a finale.

"Linear Algebra and Geometry" by Kostrikin & Manin is an excellent advanced book bridging Linear Algebra to many advanced topics including Lie algebras, category theory, Clifford algebras, affine and projective geometry, tensors and multilinear algebra, and many applications in quantum mechanics and physics.

Advanced Calculus Edit

The term *Advanced Calculus* has come to mean different things over the course of the past century. During the first half of the 20th century, *Advanced Calculus* courses consisted of what's now commonly found in Multivariable and Vector Calculus possibly with some Differential Equations topics thrown in. Lately, it has been fashionable to call very watered down "Real Analysis" courses *Advanced Calculus* even though it's not *advanced* nor *calculus* and goes no deeper into analysis than a good rigorous calculus book does. Here Advanced Calculus means what the name implies, advanced topics in calculus (and tools from analysis) typically not found in the usual calculus sequence but still very useful for solving difficult problems in science, engineering, and mathematics.

Complex Variables Edit

Complex Variables (also known as *Complex Calculus* or *Applied Complex Analysis*) is the generalization of calculus over the complex field and shares many parallels with multivariable/vector calculus.

- A First Course in Complex Analysis with Applications by Zill and Shanahan
- Fundamentals of Complex Analysis: With Applications to Engineering and Science by Saff and Snider
- Complex Variables: Introduction and Applications by Ablowitz and Fokas [Errata]
- Functions of a Complex Variable: Theory and Technique by Carrier, Krook, and Pearson

A good supplement to any of the above is Visual Complex Analysis by Needham. Once you've finished a book on complex variables, Conformal Mapping: Methods and Applications (Dover Books) by Schinzinger and Laura makes a nice supplement with many applications of conformal mapping in MechE/AeroE and EE.

Special Functions Edit

Special Functions used to be the subject of a second semester complex variables course until it was sucked into Mathematical Physics, Advanced Engineering Mathematics and other similar courses. The problem with such courses is that they spend far too little time developing subject as they try to cover complex variables, PDEs, differential geometry, topology, variations, algebra, and numerical methods among other subjects at the same time. The following books give a more focused and fuller development of special functions:

- Special Functions & Their Applications (Dover Books) by Lebedev
- Special Functions for Scientists and Engineers (Dover Books) by Bell
- The Functions of Mathematical Physics (Dover Books) by Hochstadt

For books with more mathematical theory:

- Special Functions by Andrews, Askey, and Roy
- A Course of Modern Analysis by Whittaker and Watson

- Special Functions by X. Z. Wang and Guo (Great complement to Whittaker and Watson)

Whittaker and Watson has been the bible for special functions for over a century now. Part 1 contains a review of the essential real and complex analysis needed for Part 2 which details the major special functions.

Fourier Transforms Edit

The Fourier transform and related transforms are powerful techniques used throughout STEM that convert a function into its frequency components. Tragically, many science and engineering programs can't find room for such a course in their curricula and try to get away with throwing in brief discussion of how to use them into the courses that require them. This in the end fails to create any conceptual understanding of what's going on beyond the mindless crank turning. These books will help you see the Fourier transform beyond just a 'trick' and be better equipped to apply them:

- The Fourier Transform & Its Applications by Bracewell (Great for conceptual understanding)
- Fourier Transforms: An Introduction for Engineers by Gray and Goodman
- Linear Systems, Fourier Transforms, and Optics by Gaskill
- A First Course in Fourier Analysis by Kammler

The subject matter overlaps considerably with EEE's Systems and Signals books. For more mathematical detailed books see the Fourier Analysis books below.

Calculus of Variations Edit

Calculus of Variations is the subject of finding *functions* that maximize or minimize some equation. For example, finding a path that minimizes the distance traveled from point a to b.

- Calculus of Variations (Dover Books) by Gelfand and Fomin
- Calculus of Variations: with Applications to Physics and Engineering (Dover Books) by Weinstock
- Calculus of Variations (Dover Books) by Elsgolc

Integral Equations Edit

- Introduction to Integral Equations with Applications by Jerri
- The Classical Theory of Integral Equations: A Concise Treatment by Zemyan
- Integral Equations (Dover Books) by Tricomi
- Linear Integral Equations by Kress
- Singular Integral Equations: Boundary Problems of Function Theory and Their Application to Mathematical Physics (Dover Books) by Muskhelishvili (more advanced)

Asymptotics Edit

- Asymptotic Methods in Analysis (Dover Books on Mathematics) by de Bruijn
- Asymptotic Expansions (Dover Books on Mathematics) by Erdelyi
- Asymptotic Expansions of Integrals (Dover Books on Mathematics) by Bleistein and Handelsman
- Asymptotics and Special Functions by Olver

Perturbation Theory Edit

- A First Look at Perturbation Theory (Dover Books on Physics) by Simmonds and Mann Jr (Primmer)
- Introduction to Perturbation Methods by Holmes
- Advanced Mathematical Methods for Scientists and Engineers: Asymptotic Methods and Perturbation Theory by Bender and Orszag
- Perturbations: Theory and Methods by Murdock
- Perturbation Theory for Linear Operators by Tosio Kato

Nonlinear Dynamics and Chaos Edit

- Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering by Strogatz [Author's online course]
- Dynamical Systems (Dover Books on Mathematics) by Shlomo Sternberg [Free]
- Introduction to Applied Nonlinear Dynamical Systems and Chaos by Wiggins

Obscure Topics Edit

Integration Techniques Edit

If you have read about Feynman, you may have heard his story of coming across Advanced Calculus by Woods and discovering the differentiating parameters under the integral sign[1][2][3] trick and using it to his advantage over and over again. These books are in a similar vein, they contain many integration techniques like $\int f^{-1}(x)\,dx = xf^{-1}(x) - F(f^{-1}(x))$ that aren't covered in a standard single variable calculus course but are accessible to anyone who's completed the course.

- Irresistible Integrals: Symbolics, Analysis and Experiments in the Evaluation of Integrals by Boros and Moll
- Inside Interesting Integrals: (with an introduction to contour integration) A Collection of Sneaky Tricks, Sly Substitutions, and Numerous Other Stupendously Clever, Awesomely Wicked, and Devilishly Seductive Maneuvers for Computing Nearly 200 Perplexing Definite Integrals From Physics, Engineering, and Mathematics (Plus 60

Challenge Problems with Complete, Detailed Solutions) by Nahin

Fractional Calculus Edit

Just as you can iterate to get second derivatives and triple integrals, it's possible to extend the order of these operators from integers to fractions or to any real or complex number. For example, you can define a half derivative operator where if you apply it twice to a function, you get the usual derivative of that function. This is the domain of Fractional Calculus which has a wide variety of applications in many branches of physics and engineering. The idea of fractional calculus is an old one dating back to Leibniz in 1695 and its applications were examined by the electrical engineer Oliver Heaviside in the 1890s but the first textbook on the subject was only published in 1974 by Oldham and Spanier. Since then fractional calculus has steadily been gaining more attention but it still remains relatively unknown to many in the STEM field.

- The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order (Dover Books) by Oldham and Spanier
- An Introduction to the Fractional Calculus and Fractional Differential Equations by Miller and Ross

Partial Differential Equations Edit

Historically, the study of PDEs was a major impetus for the development of many results of analysis. Without this advanced math knowledge, the study of PDE is destined to be somewhat more trickier than what you've seen before in your studies. Be prepared to do some real work.

For a quick primer on PDEs, "Partial Differential Equations for Scientists and Engineers" (Dover Books) by Farlow is pretty good albeit somewhat shallow. Fuller undergraduate treatments can be found with:

- Applied Partial Differential Equations: With Fourier Series and Boundary Value Problems by Haberman (The best applied text at this level)
- Partial Differential Equation: An Introduction by Strauss
- Partial Differential Equations by Fritz John
- Partial Differential Equations: Theory and Technique by Carrier and Pearson (Excellent source of insightful challenging problems)

Graduate Partial Differential Equations Edit

Once you have the required background in analysis, you can really study the meat of PDEs in detail with the following:

- Partial Differential Equations by Jost (Strong bias for elliptic equations)
- Partial Differential Equations by Evans (The standard introduction text for graduate PDEs)
- Introduction to Partial Differential Equations by Folland (An more intermediate graduate level PDEs book than Evans)
- Partial Differential Equations I: Basic Theory; II: Qualitative Studies of Linear Equations; III: Nonlinear Equations by Taylor

Numerical Analysis Edit

Survey of Numerical Analysis

- A Theoretical Introduction to Numerical Analysis by Ryaben'kii and Tsynkov
- A First Course in Numerical Analysis (Dover Books) by Ralston and Rabinowitz

See also the CS&E pages on

- Overviews of Numerical Analysis
- Numerical Linear Algebra
- Approximation Theory
- Numerical Ordinary Differential Equations
- Finite Difference Methods
- Finite Element Methods
- Spectral Methods

Proofs and Mathematical Reasoning Edit

True mathematics involves proofs, lots and lots of proofs (cry more physicists). The importance of mastering the art of writing valid proofs that do not make careless unstated assumptions or unproven assertions can not be understated. Oftentimes when you view some statement as initially obvious, it will turn out to be either dead wrong or at the very least hold most of the meat of the proof in proving it. Another aspect in learning proofs is following along when reading a proof in mathematical texts which requires diligently filling in *all* the skipped steps and checking which assumptions could be removed/weakened or what fails when they are removed/weakened. At their core, basic proofs are really easy and frequently just a matter of unwrapping the definition and following your nose, but getting into the right mindset for them might take the neophyte some practice in order to see them that way. Therefore you should work through a book or two on proofs before moving onto advanced mathematics and then blaming those books for being written badly because you lacked the prerequisite mathematical maturity from skipping this step. Some good books to learn proofs are:

- A Transition to Advanced Mathematics by Smith, Eggen, and St. Andre
- A Primer of Abstract Mathematics by Ash
- Conjecture and Proof by Laczkovich (An excellent supplement to either of the above books that shows a larger variety of proofs in mathematics)
- Proofs from THE BOOK by Aigner and Ziegler (Not a textbook on proofs but it is an excellent collection of well done and elegant proofs to appreciate and draw inspiration from)

If you find yourself struggling with proofs, then the following books provide more hand holding on the subject (but at the cost of excluding some additional material):

- How to Prove It: A Structured Approach by Velleman
- How to Read and Do Proofs: An Introduction to Mathematical Thought Processes by Solow
- Book of Proof by Hammack

After this, set theory and mathematical logic are the logical continuation of this material and reading books on them will deepen your understand of what sets and proofs *really are* as well as mathematics as a whole with meta-mathematics. They also make excellent next steps in getting better at proofs and abstract mathematics in general before moving on to the much more difficult subjects like algebra and analysis. Combinatorics, graph theory, linear algebra involving vector spaces, and number theory textbooks would then be the next level to practice on and are fairly easy to read at this stage of mathematical maturity.

Since you will likely find yourself revising your proofs quite often, now would be an ideal time to finally learn LaTeX (pronounced "lay-tech") to typeset your proofs and future papers in.

Set Theory, Mathematical Logic, and MetaMathematics Edit

This is the formal study of the Foundations of Mathematics using mathematics, particularly on Set Theory which much of mathematics is built on and Mathematical Logic which studies what proofs are and the limits of what can be done. When starting in this subject the question of where to start pops up. Ideally, you would want to know a some logic while studying set theory and know some set theory while studying logic leading to a bit of a dilemma. This is solved by most introductory books giving just enough material on the other subject so you don't get lost but once you move on to intermediate and beyond books, you are assumed to have already studied both set theory and logic at least at the introductory level.

Introductory Set Theory Edit

- Elements of Set Theory by Enderton

Enderton is a gentle, clear, and easy to read textbook that's perfect for someone just finishing a book or course on proofs and looking for the next step to improve their math skills further. He will construct the real numbers from ZF axioms in the first five chapters.

- The Joy of Sets: Fundamentals of Contemporary Set Theory by Devlin
- Introduction to Set Theory by Hrbacek and Jech (baby Jech)

These books would be better for someone who already has a few proof based math courses under their belt. They're a notch harder than Enderton and go into a few more advanced topics too.

Introductory Logic Edit

- Introduction to Logic: and to the Methodology of Deductive Sciences (Dover Books) by Alfred Tarski
- Introduction to Metamathematics by Kleene
- A Mathematical Introduction to Logic by Enderton (gold standard)

Intermediate Set Theory and Logic Edit

- Set Theory: An Introduction to Independence Proofs by Kunen (The newer edition is just called "Set Theory" but still is focused on independence proofs)
- Model Theory: An Introduction by Marker (have had a good course in algebra)
- Fundamentals of Mathematical Logic by Hinman
- Computability Theory by Cooper (overlaps with chapter 3 of Enderton Logic)

Advanced Computability (Recursion) Theory Edit

- Recursively Enumerable Sets and Degrees by Soare
- Computable Structures and the Hyperarithmetical Hierarchy by Ash and Knight
- Higher Recursion Theory by Sacks

Advanced Set Theory Edit

- Set Theory by Jech (More of a reference book)
- The Higher Infinite: Large Cardinals in Set Theory from Their Beginnings by Kanamori

- Constructibility by Devlin (there are errors be careful look up on SE first)[Free]
- Descriptive Set Theory by Moschovakis (getting dated)

Number Theory

Edit

- An Introduction to the Theory of Numbers by Niven, Zuckerman, and Montgomery (This book will make you into a number theorist)
- An Introduction to the Theory of Numbers by Hardy and Wright
- Number Theory (Dover Books) by Andrews (The first half is a combinatorial approach to basic number theory followed by an introduction to combinatorial number theory by a pioneer in the field)

More advanced books:

- A Classical Introduction to Modern Number Theory by Ireland and Rosen (A bit harder than the above, the goto book after you've taken undergrad abstract algebra)
- A Course in Arithmetic by Serre (Much harder than the above, a good second book)

Analytic Number Theory

Edit

- Introduction to Analytic Number Theory by Apostol
- Modular Functions and Dirichlet Series in Number Theory by Apostol
- Multiplicative Number Theory by Davenport
- Multiplicative Number Theory I: Classical Theory by Montgomery and Vaughan
- Analytic Number Theory by Iwaniec and Kowalski (The reference)

Algebraic Number Theory

Edit

- Primes of the Form x^2+ny^2: Fermat, Class Field Theory, and Complex Multiplication by Cox
- Number Theory 1: Fermat's Dream; 2: Introduction to Class Field Theory; 3: Iwasawa Theory and Modular Forms by Kazuya Kato, Nobushige Kurokawa, and Takeshi Saito
- Algebraic Number Theory and Fermat's Last Theorem by Ian Stewart and Tall
- Algebraic Number Theory by Cassels and Fröhlich
- Algebraic Number Theory by Neukirch and Schappacher

Computational Number Theory

Edit

- A Computational Introduction to Number Theory and Algebra by Shoup
- Prime Numbers: A Computational Perspective by Crandall and Pomerance
- Algorithmic Number Theory: Lattices, Number Fields, Curves and Cryptography by Buhler and Stevenhagen
- A Course in Computational Algebraic Number Theory by Cohen
- Advanced Topics in Computational Number Theory by Cohen

Elliptic Curves

Edit

- Rational Points on Elliptic Curves by Silverman and Tate
- The Arithmetic of Elliptic Curves by Silverman
- Advanced Topics in the Arithmetic of Elliptic Curves by Silverman

Probability and Randomness

Edit

Probability (Multivariable Calculus based)

Edit

- Introduction to Probability by Bertsekas and Tsitsiklis
- Introduction to Probability by Blitzstein and Hwang
- First Course in Probability by Ross
- Introduction to Probability Theory by Hoel, Port, and Stone
- The Probability Tutoring Book: An Intuitive Course for Engineers and Scientists (and Everyone Else!) by Ash

Stochastic Processes

Edit

- Introduction to Stochastic Processes by Hoel, Port, and Stone
- Introduction to Probability Models by Ross
- Stochastic Processes by Ross
- A First Course in Stochastic Processes by Karlin and Taylor
- A Second Course in Stochastic Processes by Karlin and Taylor
- Probability and Random Processes by Grimmett and Stirzaker
- Stochastic Processes in Physics and Chemistry by Van Kampen

Mathematical Statistics

Edit

See the universal recommendations on Statistics.

Design of Experiments Edit

See the universal recommendations on Design of Experiments.

Measure Theoretic Probability Theory Edit

- Probability with Martingales by David Williams
- Probability and Measure by Billingsley
- An Introduction to Probability Theory and Its Applications Vol. 2 by Feller
- Probability Theory: A Comprehensive Course by Klenke
- Probability-1 by Shiryaev

Stochastic Calculus Edit

- Stochastic Differential Equations: An Introduction with Applications by Bernt Øksendal
- Diffusions, Markov Processes and Martingales: Volume 1 Foundations; Volume 2 Itô Calculus by Rogers and Williams
- Brownian Motion and Stochastic Calculus by Ioannis Karatzas and Steven Shreve

Combinatorics and Graph Theory Edit

Primers in Combinatorics and Graph Theory Edit

These 2 books are aimed at high school students with knowledge of elementary algebra to give them a taste of pure mathematics.

- Mathematics of Choice: Or, How to Count Without Counting by Ivan Niven
- Introduction to Graph Theory (Dover Books) by Trudeau

Introduction to Combinatorics Edit

- Combinatorics: Topics, Techniques, Algorithms by Cameron [Website]
- A Course in Combinatorics by van Lint and Wilson

Introduction to Graph Theory Edit

- Graph Theory by Diestel
- Modern Graph Theory by Bollobas

Enumerative Combinatorics Edit

- A Course in Enumeration by Aigner
- Enumerative Combinatorics: Volume 1&2 by Stanley [Website]

Extremal Combinatorics and Graph Theory Edit

- Extremal Combinatorics: With Applications in Computer Science by Jukna [Website]
- Extremal Graph Theory (Dover Books) by Bollobas
- The Probabilistic Method by Alon and Spencer

Algebraic Graph Theory Edit

- Algebraic Graph Theory by Biggs
- Algebraic Graph Theory by Godsil and Royle [Website]

Linear Algebraic Graph Theory Edit

- Graphs and Matrices by Bapat
- Graph Spectra by Brouwer and Haemers [Draft]
- An Introduction to the Theory of Graph Spectra by Cvetković, Rowlinson, and Simić

See also Combinatorial Optimization & Network Flows and Combinatorial Game Theory on the CS&E page.

Abstract Algebra Edit

Abstract Algebra (also called *Modern Algebra* or just *Algebra*) is the study of mathematical structures that consist of a set with algebraic rules defined on the set's elements. This enables us to prove general results that depend only on the particular rules the structures have and not a particular example structure (like the rationals, reals, quaternions, polynomials, matrices, or integers modulo n) we have in mind that satisfies those rules.

Abstract Algebra is not to be mistaken with *College Algebra* as that refers to the Elementary Algebra that is typically done in grade school. It's called *College Algebra* because, well, nobody would pay for a course called *The Algebra You Should Have Learned in High School But Were Too Much Of A Fuck Up To Do So*. If you're looking for resources on that, see the Precalculus section above.

Group Theory Teaser Edit

These books are accessible enough to give freshmen or high school students a digestible taste of abstract math and build intuition for when they later get to Algebra

- Groups and Their Graphs by Grossman and Magnus (sadly out of print)
- Visual Group Theory by Carter
- Groups and Symmetry by Armstrong

First Year Algebra (Undergrad) Edit

- Algebra by Artin
- Topics in Algebra by Herstein (Herstein's Abstract Algebra is an abbreviated version of his Topics book for a one semester course)
- Abstract Algebra by Dummit and Foote (more of an encyclopedic reference than a book)
- Algebra by Mac Lane and Birkhoff
- Abstract Algebra by Grillet

Herstein's Topics takes a fairly conventional approach to the subject while Artin's book does things in a rather unique and geometrical way. While both are well written texts on their own, but pairing them is very useful.

Survey of Abstract Algebra Applications Edit

- Applied Abstract Algebra by Lidl and Pilz
- Applications of Abstract Algebra with Maple and Matlab by Klima, Sigmon, and Stitzinger
- Topics in Applied Abstract Algebra by Nagpaul and Jain

Matrix Groups Edit

- Matrix Groups for Undergraduates by Tapp
- Naive Lie Theory by Stillwell

Second Year Algebra (Graduate) Edit

- Basic Algebra I & II (Dover Books) by Jacobson (Vol I is closer to 1st year books in terms of level)
- Algebra by Hungerford (Don't mistake it for his "Abstract Algebra" book which is on a lower level)
- Algebra by Lang [A Companion to Lang's Algebra]

Commutative Algebra Edit

- Introduction to Commutative Algebra by Atiyah and Macdonald
- Commutative Algebra with a View Towards Algebraic Geometry by Eisenbuds

Group Theory Edit

- An Introduction to the Theory of Groups by Rotman
- Finite Group Theory by Isaacs
- A Course in the Theory of Groups by Robinson

Analysis Edit

Mathematical Analysis' origins are found in the age old struggle of mathematicians to deal with the infinite and infinitesimal dating all the way back to Eudoxus of Cnidus and Archimedes of antiquity. With the piecemeal development of calculus by Cavalieri, Pascal, Fermat, Descarte, Leibniz, Euler, Lagrange, Fourier and many others, calculus gradually showed itself to be a powerful yet deeply troubled tool. As much as mathematicians tried, they struggled with clearly defining key stumbling points: the concept of an infinitesimal number smaller than 1/n for all integers n yet nonzero in a logically consistent manner, the concept of infinite approach, division by 0, and the rules in which an infinite series may be manipulated and examined. These were not just pointless trifles that could be brushed off as more philosophy than math but of increasing practical importance. As time went on, many counterexamples (and not just pathological ones) where the naive application of the methods of calculus would produce erroneous results cast a shadow on the validity of all other results of calculus. Many critics wanted to end its study altogether and relentlessly mocked the concept of infinitesimals as "the ghosts of departed quantities". Since the triumphs for calculus were both numerous and far reaching, mathematicians strongly sought to make the results of calculus proven rather than discarding the subject all together. This situation was finally resolved only in the early 19th century with the work of Cauchy and Weierstrass and the ε-δ definition of a limit (which would ironically kill off infinitesimals until the 1960s and the advent nonstandard analysis) that birthed the new field of analysis. This sparked off a massive revolution in mathematics and the field of analysis quickly exploded into various distinct but interconnected directions.

Students just finishing the study of calculus and basic proofs often fail to realize the sheer importance of careful work in analysis and scoff the whole subject off as merely "intellectual or autistic masturbation". This mentality comes from being coddled with the toy-problems you see in calculus that are selected to hide any possible nastiness that comes from complicated situations that frequently arise in science and engineering. Even if the student is aware of the importance of being careful, they are often insulted when forced to work through "obvious" theorems. The problem here is that many results in analysis that seem obvious are frankly very difficult to prove (for example see the Jordan curve theorem) or even dead wrong. In order to gain the ability to prove important and powerful theorems hidden away in analysis, students need plenty of practice working through basic problems to gain familiarity and mathematical maturity to move on to difficult work even if this means you need to spend time proving that "every open ball is open".

A good reference to keep with you and refer to often is "Counterexamples in Analysis" by Gelbaum and Olmsted published by Dover Books.

Inequalities

A lot of the exercises in analysis often boil down to spamming the triangle inequality until you get the result you want. If you haven't done much work with inequalities since grade school, practicing them can make the subject seem vastly easier.

- The Cauchy-Schwarz Master Class: An Introduction to the Art of Mathematical Inequalities by Steele
- Inequalities by Hardy, Littlewood, and Polya

Real Analysis (Metric Space based)

This is where the results of single variable calculus are finally made both rigorous and generalized.

The gold standard for the subject is the first 8 chapters of Rudin's "Principles of Mathematical Analysis" whose slick proofs and challenging exercises can't be beat. Chapters 9 and 10 of Rudin on multivariable analysis are bit sparse to learn from so you're better off moving fuller treatment on analysis on manifolds (see below) to learn from. The final chapter 11 is completely skippable as other books are far better in their treatment of Lebesgue integrals/measure theory than Rudin's brief survey. Apostol's "Mathematical Analysis" goes through a bit more material than Rudin, gives more worked out proofs, and has relatively easier problems. If you're struggling with Rudin, give Apostol a try. Zorich's "Mathematical Analysis I && II" starts lower level than Rudin but ends on much higher level covering many additional topics including manifolds. The price paid is that his books is quite longer than Rudin and larger time investment.

Analysis on Manifolds

This is the study of analysis on multidimensional spaces making multivariate and vector calculus rigorous and pushing the subject further. A good grounding in linear algebra is required.

- Munkres' Analysis on Manifolds
- Spivak's Calculus on Manifolds
- do Carmo's Differential Forms and Applications

Fourier Analysis

- Fourier Series (Dover) by Tolstov
- Fourier Analysis: An Introduction by Stein & Shakarchi
- Fourier Analysis and its Applications by Folland
- Fourier Analysis by Körner
- Fourier Series and Integrals by Dym and McKean

Complex Analysis

- Complex Analysis by Stein & Shakarchi
- Functions of One Complex Variable by Conway
- Complex Analysis by Ahlfors

Graduate Real Analysis

- Real Analysis: Measure Theory, Integration, and Hilbert Spaces by Stein & Shakarchi
- Real Analysis by Royden
- Real Analysis: Modern Techniques and Their Applications by Folland
- Real and Complex Analysis by Rudin

Functional Analysis

- Functional Analysis: Introduction to Further Topics in Analysis by Stein & Shakarchi
- Functional Analysis by Lax (Includes several historic notes of the subject and the fate of its researchers during WWII)
- Functional Analysis by Rudin

Abstract Harmonic Analysis

- Introduction to Abstract Harmonic Analysis (Dover Books) Loomis
- Fourier Analysis on Groups (Dover Books) by Walter Rudin
- A Course in Abstract Harmonic Analysis by Folland

Nonstandard Analysis

- Nonstandard Analysis (Dover Books) by Robert
- Applied Nonstandard Analysis (Dover Books) by Martin Davis
- Nonstandard Analysis: Theory and Applications by Arkeryd, Cutland and Henson
- Lectures on the Hyperreals: An Introduction to Nonstandard Analysis by Goldblatt

- Nonstandard Analysis for the Working Mathematician by Loeb and Wolff
- Nonstandard Methods in Stochastic Analysis and Mathematical Physics (Dover Books) by Albeverio, Hoegh-Krohn, Fenstad, and Lindstrom
- Non-standard Analysis by Abraham Robinson (The original book on the subject)

Topology Edit

Point-set Topology Edit

- Topology by James Munkres (The standard for most topology courses)
- Introduction to Topological Manifolds by John Lee
- Elementary Topology (Dover Books) by Michael Gemignani
- General Topology (Dover Books) by Stephen Willard (A bit more difficult than the above)

A great supplement to any General Topology book is "Counterexamples in Topology" (Dover Books) by Steen and Seebach.

Algebraic Topology Edit

- Algebraic Topology by Hatcher
- Differential Forms in Algebraic Topology by Bott and Tu
- A Concise Course in Algebraic Topology by May (Advanced)

Differential Topology Edit

- Topology from the Differential Viewpoint by Milnor
- Differential Topology by Victor Guillemin and Alan Pollack
- Differential Topology by Hirsch (More advanced)

Knot Theory Edit

- "Knot Theory" by Livingston (Primmer)
- "Knots and Physics"; "On Knots"; "Formal Knot Theory" (Dover Books) by Kauffman
- "An Introduction to Knot Theory" by Lickorish
- "Knots and Links" by Rolfsen
- "Knot Theory and Its Applications", "A Study of Braids" by Kunio Murasugi
- "Knots, Links, Braids and 3-Manifolds: An Introduction to the New Invariants in Low-Dimensional Topology" by Prasolov and Sossinsky

Geometry Edit

Smooth Manifolds Edit

- Introduction to Smooth Manifolds by John Lee
- An Introduction to Manifolds by Loring W. Tu
- A Comprehensive Introduction to Differential Geometry 1 by Spivak

Riemann Geometry Edit

- Riemannian Manifolds: An Introduction to Curvature by John Lee
- Riemannian Geometry by do Carmo
- Riemannian Geometry and Geometric Analysis by Jost

Algebraic Geometry Edit

Primers in Algebraic Geometry:

- Basic Algebraic Geometry 1: Varieties in Projective Space by Shafarevich
- Basic Algebraic Geometry 2: Schemes and Complex Manifolds by Shafarevich
- An Invitation to Algebraic Geometry by Karen Smith, Lauri Kahanpää, Pekka Kekäläinen, and William Traves
- The Geometry of Schemes (Graduate Texts in Mathematics) by Eisenbud, and Harris

Some more serious stuff:

- Algebraic Geometry (Graduate Texts in Mathematics) by Robin Hartshorne (The standard)
- Principles of Algebraic Geometry by Griffiths and Harris (Complex Geometry)